WAVE MOTION

&

SOUND

Contents

Wave Motion

When a particle moves through space, it carries kinetic energy with itself. The energy is associated with the particle and is transported from one region of the space to other.

Energy can also be transported from one point in space to another without any bulk motion of material.

> e.g. When we say something, it creates a disturbance in the air close to our lips. The air particles are either pulled or pushed. Hence, the density in that region increases or decreases temporarily. This disturbed layer exerts a force on the next layer and thus the disturbance travels to the ear of the listener. Neither the speaker nor the listener moves. Even the intervening air also does not move.

This type of motion of energy is called wave motion. Hence, we can define wave motion as,

> *Wave motion is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean positions, the disturbance being handed on from one particle to the next particle in the medium.*

Types of Waves

Waves can be distinguished in two ways,

a) <u>On the basis of medium required for propagation:</u>
   i) Waves that require a medium for propagation are called mechanical waves. e.g. Sound waves and water waves.
   ii) Waves that don't require a medium for propagation are called non – mechanical waves. e.g. Electromagnetic radiation.

b) <u>On the basis of direction of vibration of particles about their mean position:</u>
   i) A wave that causes the particles of the disturbed medium to move perpendicular to the direction of wave motion about their mean position is called a transverse wave.
   ii) A wave that causes the particles of the disturbed medium to move parallel to the direction of wave motion about their mean position is called a longitudinal wave.

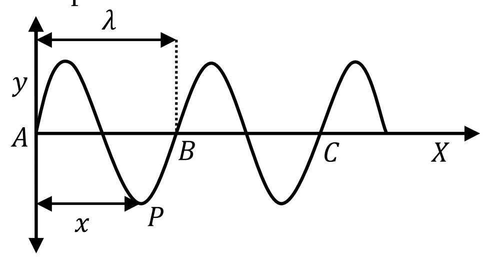
Characteristics of Wave Motion

The characteristics of wave motion are,

a) Wave is a form of disturbance which propagates in a medium.
b) When the wave propagates, particles of the medium oscillate about their mean positions.
c) There is a definite phase difference between two consecutive particles.
d) The velocity of the wave is different from the velocity of the particles. The velocity of wave is a constant for a given medium but the velocity of the particle goes on changing, being maximum in the mean position and minimum in the extreme position.
e) Wave motion is possible only in inertial and elastic media.
f) At extreme positions, energy is wholly potential and at mean positions is wholly kinetic.
g) When a wave travels through a medium, there is a flow of energy without any transfer of matter.

Equation of Wave Motion

In wave motion, particles vibrate about their mean positions either perpendicular or parallel to the direction of the wave motion. The acceleration produced is always directed towards the mean position and varies as its distance from the mean position. Hence, the particles of medium execute simple harmonic motion about their mean positions.



Let us consider a transverse wave being propagated in a string from left to right along the positive X – direction. The equation of motion of any particle, say A, is given as

$$y = a \sin \omega t$$

where, a is the amplitude of the vibrating particle, y the displacement after a time t and ω the angular velocity.

If n is the frequency of the vibration, then $\omega = 2\pi n$ and the displacement y is given as,

$$y = a \sin 2\pi n t$$

The direction of propagation of particles B and C passing through their mean positions is same as that of particle A. Hence, particles A, B and C are said to be in same phase. The distance AB between two consecutive particles in the same phase is called the wavelength and is denoted by λ. The phase changes by 2π in going from A to B.

Hence, in going from A to any point P at a distance x from A, the phase changes by ϕ. This phase difference is given as,

$$\phi = \frac{2\pi}{\lambda} x$$

Hence, the displacement y is given by,

$$y = a \sin(\omega t - \phi)$$

$$= a \sin\left(2\pi n t - \frac{2\pi}{\lambda} x\right)$$

$$= a \sin\left(2\pi \frac{v}{\lambda} t - \frac{2\pi}{\lambda} x\right)$$

$$= a \sin \frac{2\pi}{\lambda}(vt - x)$$

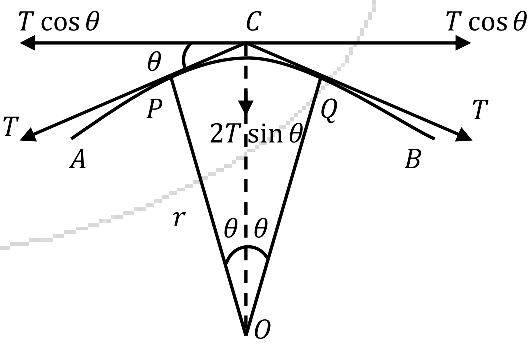
The quantity $\frac{2\pi}{\lambda} = k$ is called the propagation constant. Hence, we get

$$y = a \sin(\omega t - kx)$$

This is the general equation of a particle of a wave motion.

Velocity of Transverse Waves Produced in a String

A string is a cord whose length is very large as compared to its diameter and is perfectly uniform and flexible. When such a string is stretched between two points and is plucked perpendicular to its length, transverse waves are produced in it.



Let us consider a part of wave AB travelling in the string left to right with velocity v. If the string is pulled from right to left with the same velocity, the wave AB will remain stationary with respect to the paper in space. A small part PQ of the wave AB can be considered to be arc of a circle centered at O. Let the radius of the circle be r. As the string moves along the circular arc PQ, a centripetal force acts on it along the radius r. A uniform tension T acts in the string throughout. This tension T acts along CP at P and along CQ at Q, where CP and CQ are tangents to the arc PQ at P and Q respectively.

Let the angle POQ be denoted by 2θ. According to the geometry of the figure, arc PQ is given as,

$$Arc\ PQ = 2r\theta$$

Let ρ the mass per unit length of the string, then the mass of the part $PQ = 2\rho r\theta$

The centripetal force acting on part PQ is given as,

$$= (2\rho r\theta)\frac{v^2}{r} = 2\rho\theta v^2$$

The tension T is resolved into two rectangular components. The horizontal component $T cos\,\theta$ acting along CM and CN, cancel each other being equal and opposite. The vertical components $T\sin\theta$ acts in the same direction and adds up along CO. It is given as,

$$= 2T\sin\theta = 2T\theta, \quad \text{as } \theta \text{ is very small, } \sin\theta \approx \theta.$$

This tension provides the necessary centripetal force. Hence, we get

$$2T\theta = 2\rho\theta v^2$$

$$\Rightarrow T = \rho v^2$$

Hence, velocity v of the transverse waves is given as,

$$v = \sqrt{\frac{T}{\rho}}$$

Interference of Waves

When two or more waves simultaneously pass through a point, the disturbance at the point is given by the algebraic sum of the disturbances each wave would produce in absence of the other wave(s). When such superposition happens, we say interference of waves have taken place.

Let us consider two sinusoidal waves travelling in the same direction having the phase difference, ϕ. Let the equations of the wave be given as,

$$y_1 = a_1 \sin(\omega t - kx)$$
$$y_2 = a_2 \sin(\omega t - kx + \phi)$$

According to principle of superposition, the resultant wave is given as

$$y = y_1 + y_2$$
$$= a_1 \sin(\omega t - kx) + a_2 \sin(\omega t - kx + \phi)$$
$$= a_1 \sin(\omega t - kx) + a_2 \sin(\omega t - kx) \cos\phi + a_2 \cos(\omega t - kx) \sin\phi$$
$$= (a_1 + a_2 \cos\phi) \sin(\omega t - kx) + a_2 \sin\phi \cos(\omega t - kx)$$

Let us define,

$$a_1 + a_2 \cos\phi = a \cos\theta$$
$$a_2 \sin\phi = a \sin\theta$$

Therefore we get,

$$\tan\theta = \frac{a \sin\theta}{a \cos\theta} = \frac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$$

$$a = \sqrt{(a\cos\theta)^2 + (a\sin\theta)^2}$$
$$= \sqrt{(a_1 + a_2 \cos\phi)^2 + (a_2 \sin\phi)^2}$$
$$= \sqrt{a_1^2 + a_2^2 cos^2\phi + 2a_1a_2 \cos\phi + a_2^2 sin^2\phi}$$

Hence, the resultant wave is given as,

$$y = a \cos\theta \sin(\omega t - kx) + a \sin\theta \cos(\omega t - kx)$$
$$y = a \sin(\omega t - kx + \theta)$$

Hence, the resultant wave train has same frequency but different amplitude a and phase θ. Let us consider two cases,

a) Phase difference is zero:

When the phase difference between the two waves is zero,

the amplitude of the resultant wave is given as, $a = a_1 + a_2$

and the phase difference, $\tan\theta = 0 \Longrightarrow \theta = 0$

Thus the waves reinforce each other and the amplitude is the sum of the amplitudes of the component waves. The resultant vibration is in phase with the component vibrations.

b) Phase difference is 180^0:

When the phase difference between the two waves is 180^0,

the amplitude of the resultant wave is given as, $a = a_1 - a_2$. If $a_1 = a_2$, then $a = 0$.

and the phase difference, $\tan\theta = 0 \Longrightarrow \theta = 0$

Thus the waves destroy each other's effect and resultant amplitude is difference of the component waves. The resultant vibration is in phase with the component vibrations.

BEATS

When two waves of nearly the same frequency travel along the same straight line in the same direction, the resultant displacement at a point alternately waxes and wanes in amplitude as many times per second as the difference between their frequencies. This phenomenon is called beats.

Let us consider two sinusoidal waves travelling in the same direction having frequencies n and m. Let the equations of the two waves be given as,

$$y_1 = a\sin 2\pi nt$$
$$y_2 = a\sin 2\pi mt$$

According to principle of superposition, the resultant wave is given as

$$\begin{aligned} y &= y_1 + y_2 \\ &= a\sin 2\pi nt + a\sin 2\pi mt \\ &= 2a\sin\left(\frac{2\pi nt + 2\pi mt}{2}\right)\cos\left(\frac{2\pi nt - 2\pi mt}{2}\right) \\ &= 2a\cos\left[2\pi t\left(\frac{n-m}{2}\right)\right]\sin\left[2\pi t\left(\frac{n+m}{2}\right)\right] \\ &= A\sin\left[2\pi t\left(\frac{n+m}{2}\right)\right] \end{aligned}$$

This equation represents a periodic vibration with amplitude $A = 2a\cos\left[2\pi t\left(\frac{n-m}{2}\right)\right]$ and frequency $\left(\frac{n+m}{2}\right)$, which is the arithmetic mean of the individual frequencies. The amplitude is not constant as it changes with time.

The maximum value of the amplitude is given by $A = \pm 2a$, when

$$\cos\left[2\pi t\left(\frac{n-m}{2}\right)\right] = \pm 1$$
$$\Rightarrow \pi t(n-m) = k\pi \quad \text{where, } k = 0,1,2,3,\ldots$$
$$\Rightarrow t = \frac{k}{n-m}$$
$$\Rightarrow t = 0, \frac{1}{n-m}, \frac{2}{n-m}, \frac{3}{n-m}, \ldots$$

Hence, the time interval is an integral multiple of $\frac{1}{n-m}$. The time interval between two consecutive maxima is $\frac{1}{n-m}$.

The minimum value of the amplitude is given by $A = 0$, when

$$\cos\left[2\pi t\left(\frac{n-m}{2}\right)\right] = 0$$
$$\Rightarrow \pi t(n-m) = (2k+1)\frac{\pi}{2} \quad \text{where, } k = 0,1,2,3,\ldots$$
$$\Rightarrow t = \frac{2k+1}{2(n-m)}$$
$$\Rightarrow t = \frac{1}{2(n-m)}, \frac{3}{2(n-m)}, \frac{5}{2(n-m)}, \ldots$$

Hence, the time interval is an odd multiple of $\frac{1}{2(n-m)}$. The time interval between two consecutive maxima is $\frac{1}{n-m}$. Hence, the frequency of the maxima and minima is $n-m$.

One maximum and one minimum constitute a beat. Hence, the number of beats produced is $n-m$. Hence, the intensity of the resultant sound rises and falls $n-m$ times per second.

The resultant wave is thus a simple harmonic wave whose frequency is equal to the arithmetic mean of the component frequencies and whose amplitude varies alternately from $2a$ or the sum of the amplitudes of the component waves to zero or their difference. The vibration has a frequency equal to the difference of the component frequencies.

SOUND

Sound is produced in a material medium by a vibrating source. As the vibrating source moves forward, it compresses the medium past it. This part of the medium compresses the layer next to it by collisions. The compression travels in the medium at a speed dependent on elastic and inertial properties of the medium. As the source moves back, it drags the medium and produces a rarefaction in the layer. The layer next to it is then dragged back and thus the rarefaction pulse passes forward. In this way, compression and rarefaction pulses are produced which travel in the medium.

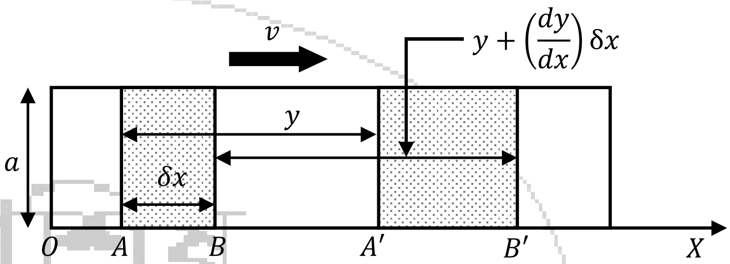
Sound waves can be categorized into three categories,

a) Audible, $20\ Hz - 20\ kHz$
b) Infrasonic, $< 20\ Hz$
c) Ultrasonic, $> 20\ kHz$

SPEED OF SOUND IN A MEDIUM

Let us consider a tube of area of cross – section a. Let us consider a sound wave to be introduced into the tube from left to right. The longitudinal sound wave produces rarefactions and compressions in the medium and travels from left to right with a velocity v along the X – axis, OX.

Let us consider two planes perpendicular to OX at A and B, a small distance δx apart. When sound waves are introduced, at any instant, the plane A be displaced to A' and plane B to the position B'. Let the displacement of the plane $A = AA' = y$ and the rate of change of displacement be $\frac{dy}{dx}$.



Hence, displacement of plane B is given as,

$$= BB' = y + \left(\frac{dy}{dx}\right)\delta x$$

As the displacement of the plane B is greater than the displacement of the plane A, the distance $B'A'$ is greater than the displacement BA by an amount given by,

$$y + \left(\frac{dy}{dx}\right)\delta x - y = \left(\frac{dy}{dx}\right)\delta x$$

Hence, the change in volume is given by,

$$= a\left(\frac{dy}{dx}\right)\delta x$$

The original volume of layer $= a\delta x$. Hence, the bulk strain is given as,

$$bulk\ strain = \frac{change\ in\ volume}{original\ volume} = \frac{a\left(\frac{dy}{dx}\right)\delta x}{a\delta x} = \left(\frac{dy}{dx}\right)$$

Let P be the pressure at the plane A. Let this pressure vary over a distance x at the rate $\frac{dP}{dx}$, then pressure at the plane B is given as $P + \left(\frac{dy}{dx}\right)\delta x$.

The resultant pressure acting on the element AB is given by,

$$P + \left(\frac{dP}{dx}\right)\delta x - P = \left(\frac{dP}{dx}\right)\delta x$$

Hence, the force acting on the element AB

$$= a\left(\frac{dP}{dx}\right)\delta x$$

Let ρ be the density of the medium. Hence, mass of the element $AB = a\rho\delta x$

According to Newton's laws of motion,

$$(a\rho\delta x)\frac{d^2y}{dt^2} = a\left(\frac{dP}{dx}\right)\delta x$$

$$\rho\frac{d^2y}{dt^2} = \frac{dP}{dx}$$

The Bulk modulus of the medium is given as,

$$K = \frac{Stress}{Volume\ Strain}$$

$$= \frac{P}{{dy}/{dx}}$$

$$\Rightarrow P = K\frac{dy}{dx}$$

$$\Rightarrow \frac{dP}{dx} = \frac{d}{dx}\left(K\frac{dy}{dx}\right) = K\frac{d^2y}{dx^2}$$

Hence, we get

$$\rho\frac{d^2y}{dt^2} = K\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dt^2} = \left(\frac{K}{\rho}\right)\frac{d^2y}{dx^2}$$

This represents the differential equation of a longitudinal (or sound) wave propagating in a medium. Comparing it with the standard equation of wave equation,

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

we get,

$$v^2 = \frac{K}{\rho}$$

$$\Rightarrow v = \sqrt{\frac{K}{\rho}}$$

This is the expression for the velocity of sound waves in an elastic medium.

Standing Waves

1. *Organ Pipe open at both ends*

An open organ pipe is a cylindrical tube open at both ends containing an air column. A source of sound near one of the open ends sends the waves in the pipe. The wave is reflected by the other open end and travels towards the source. It suffers a second reflection at the open end near the source and then interferes with the new wave sent by the source.

Two waves of the same period, wavelength, amplitude and velocity travelling along the same straight line in opposite direction superposes on each other and stationary waves are produced.

Let the equations of the two waves be,

$$y_1 = a\sin(\omega t - kx)$$

$$y_2 = a\sin(\omega t + kx)$$

These equations represent the incident wave and the reflected wave from an open end of an organ pipe.

The resultant wave equation is given as,

$$y = y_1 + y_2$$

$$= a\sin(\omega t - kx) + a\sin(\omega t + kx)$$

$$= 2a\sin\left(\frac{\omega t - kx + \omega t + kx}{2}\right)\cos\left(\frac{\omega t - kx - \omega t - kx}{2}\right)$$

$$= 2a\sin\omega t\cos kx, \qquad \because \cos(-\theta) = \cos(\theta)$$

$$y = A \sin \omega t$$

where, $A = 2a \cos kx$

The resultant wave has the period same as that of the component waves. It is also seen that amplitude varies with position (x) of the particle. The velocity of the particles due to resultant wave motion can be obtained as,

$$v = \frac{dy}{dt} = A\omega \cos \omega t$$
$$= 2a\omega \cos kx \cos \omega t$$

Nodes and Antinodes

The displacement $y = 2a \sin \omega t \cos kx$ is found to be maximum when,

$$\cos kx = \pm 1$$
$$\Rightarrow kx = 0, \pi, 2\pi, 3\pi, \ldots$$
$$\Rightarrow \frac{2\pi}{\lambda} x = 0, \pi, 2\pi, 3\pi, \ldots$$
$$\Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$$
$$\Rightarrow x = \frac{m\lambda}{2}, m = 0,1,2,3, \ldots$$

The velocity is given as $v = 2a\omega \cos kx \cos \omega t$. The velocity also varies in the same manner as the displacement and is maximum at the positions given by, $x = \frac{m\lambda}{2}, m = 0,1,2,3, \ldots$

These positions where displacement and velocity of particles is maximum are called antinodes.

The displacement $y = 2a \sin \omega t \cos kx$ is found to be minimum when,

$$\cos kx = 0$$
$$\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$\Rightarrow \frac{2\pi}{\lambda} x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$
$$\Rightarrow x = (2m+1)\frac{\lambda}{4}, m = 0,1,2,3, \ldots$$

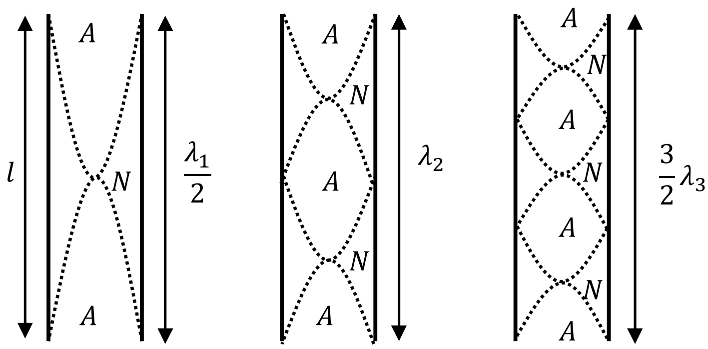
The velocity is given as $v = 2a\omega \cos kx \cos \omega t$. The velocity also varies in the same manner as the displacement and is minimum at the positions given by, $x = (2m+1)\frac{\lambda}{4}, m = 0,1,2,3, \ldots$

These positions where displacement and velocity of particles is minimum are called nodes.

Resonant Frequencies

The open ends of the organ pipe are free to communicate with the air and thus can vibrate freely. Hence, both the open ends should contain antinodes.

Two antinodes must have a node in between them. If the length of the tube is l and wavelength of the standing waves be λ_1, then the tube must contain half the wavelength. Hence, we get



$$l = \frac{\lambda_1}{2}$$

The corresponding frequency of the standing waves is given as,

$$n_0 = \frac{v}{\lambda_1} = \frac{v}{2l}$$

where, v is the velocity of the sound wave. This frequency is called the fundamental note of the standing waves. It is the first harmonic.

The next possible mode of vibration is when the tube will contain three consecutive antinodes and two nodes. Let the corresponding wavelength be λ_2. Hence, we get

$$l = \lambda_2$$

The corresponding frequency of the standing waves is given as,

$$n_1 = \frac{v}{\lambda_2} = \frac{v}{l} = 2n_0$$

This is known as the first overtone and is twice the fundamental note. It is the second harmonic.

The next possible mode of vibration is when the tube will contain four consecutive antinodes and three nodes. Let the corresponding wavelength be λ_3. Hence, we get

$$l = \frac{3}{2}\lambda_3$$

The corresponding frequency of the standing waves is given as,

$$n_2 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3n_0$$

This is known as the second overtone and is thrice the fundamental note. It is the third harmonic.

Thus the open organ pipe emits notes of frequencies $n_0, 2n_0, 3n_0, 4n_0, \ldots$ In general we can write the frequencies to be $mn_0 = \frac{mv}{2l}$, where $m = 1,2,3,4, \ldots$

All harmonics are present when air vibrates in an open pipe.

II. *Organ Pipe closed at one end*

A closed organ pipe is a cylindrical tube closed at one end containing an air column. A source of sound sends the waves in the pipe through the open end. The wave is reflected from the closed end and travels towards the source. It suffers second reflection at the open end near the source and then interferes with the new wave sent by the source.

Two waves of the same period, wavelength, amplitude and velocity travelling along the same straight line in opposite direction superposes on each other and stationary waves are produced.

Let the equations of the two waves be,

$$y_1 = a\sin(\omega t - kx)$$
$$y_2 = -a\sin(\omega t + kx)$$

These equations actually represent the incident wave and the reflected wave from the closed end of an organ pipe. The closed end of the organ pipe is a node. Hence on reflection the amplitude a changes to $-a$.

The resultant wave equation is given as,

$$y = y_1 + y_2$$
$$= a\sin(\omega t - kx) - a\sin(\omega t + kx)$$
$$= 2a\cos\left(\frac{\omega t - kx + \omega t + kx}{2}\right)\sin\left(\frac{\omega t - kx - \omega t - kx}{2}\right)$$
$$= -2a\cos\omega t\sin kx, \qquad \because \sin(-\theta) = -\sin(\theta)$$
$$y = A\ \cos\omega t$$

where, $A = -2a\sin kx$

The resultant wave has the period same as that of the component waves. It is also seen that amplitude varies with position (x) of the particle. The velocity of the particles due to resultant wave motion can be obtained as,

$$v = \frac{dy}{dt} = A\omega \sin \omega t$$
$$= 2a\omega \sin kx \sin \omega t$$

Nodes and Antinodes

The displacement $y = 2a\omega \sin kx \cos \omega t$ is found to be maximum when,

$$\sin kx = \pm 1$$
$$\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$\Rightarrow \frac{2\pi}{\lambda} x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$
$$\Rightarrow x = (2m+1)\frac{\lambda}{4}, m = 0,1,2,3, \ldots$$

The velocity is given as $v = 2a\omega \sin kx \sin \omega t$. The velocity also varies in the same manner as the displacement and is maximum at the positions given by, $x = (2m+1)\frac{\lambda}{4}, m = 0,1,2,3, \ldots$

These positions where displacement and velocity of particles is maximum are called antinodes.

The displacement $y = 2a \sin \omega t \cos kx$ is found to be minimum when,

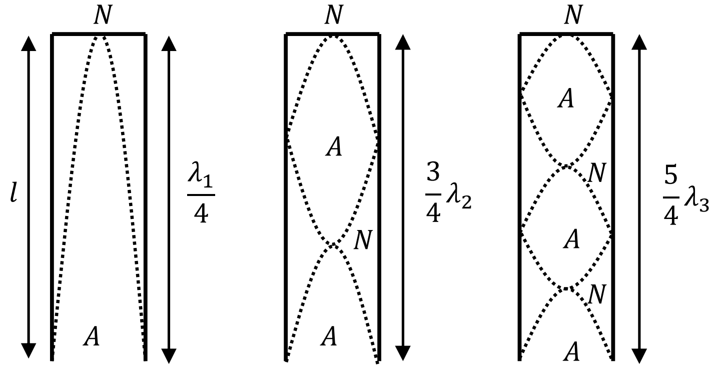
$$\text{sink}\, kx = 0$$
$$\Rightarrow kx = 0, \pi, 2\pi, 3\pi, \ldots$$
$$\Rightarrow \frac{2\pi}{\lambda} x = 0, \pi, 2\pi, 3\pi, \ldots$$
$$\Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$$
$$\Rightarrow x = \frac{m\lambda}{2}, m = 0,1,2,3, \ldots$$

The velocity is given as $v = 2a\omega \sin kx \sin \omega t$. The velocity also varies in the same manner as the displacement and is minimum at the positions given by, $x = \frac{m\lambda}{2}, m = 0,1,2,3, \ldots$ These positions where displacement and velocity of particles is minimum are called nodes.

Resonant Frequencies

The open end of the organ pipe is free to communicate with the air and thus can vibrate freely. Hence, the open end should contain an antinode. At the closed end, the layer in contact cannot move freely and a node is formed.

If the length of the tube is l and wavelength of the standing waves be λ_1, then the tube must contain a node and an antinode. Hence quarter of the wavelength will be contained in the length of the tube. Hence, we get



$$l = \frac{\lambda_1}{4}$$

The corresponding frequency of the standing waves is given as,

$$n_0 = \frac{v}{\lambda_1} = \frac{v}{4l}$$

where, v is the velocity of the sound wave. This frequency is called the fundamental note of the standing waves. It is the first harmonic.

The next possible mode of vibration is when the tube will contain two consecutive nodes and two antinodes. Let the corresponding wavelength be λ_2. Hence, we get

$$l = \frac{3}{4}\lambda_2$$

The corresponding frequency of the standing waves is given as,

$$n_1 = \frac{v}{\lambda_2} = \frac{3v}{4l} = 3n_0$$

This is known as the first overtone and is thrice the fundamental note. It is the third harmonic.

The next possible mode of vibration is when the tube will contain three consecutive nodes and three antinodes. Let the corresponding wavelength be λ_3. Hence, we get

$$l = \frac{5}{4}\lambda_3$$

The corresponding frequency of the standing waves is given as,

$$n_1 = \frac{v}{\lambda_3} = \frac{5v}{4l} = 5n_0$$

This is known as the second overtone and is five times the fundamental note. It is the fifth harmonic.

Thus the closed organ pipe emits notes of frequencies $n_0, 3n_0, 5n_0, 7n_0, \ldots$ In general we can write the frequencies to be $(2m+1)n_0 = \frac{(2m+1)v}{4l}$, where $m = 0,1,2,3,4, \ldots$

Only odd harmonics are present when air vibrates in a closed pipe.

Characteristics of Stationary Waves

a) The stationary waves are not progressive. Hence, there is no transfer of energy from one particle to other.
b) The displacement and velocity at a point continuously change from maximum positive to maximum negative value and vice versa.
c) Every particle, except at the nodes, executes simple harmonic motion with the same period as the component waves.
d) The particles at the nodes have zero displacement and zero velocity.
e) The particles at the antinodes have maximum displacement and velocity.
f) The distance between two consecutive nodes or antinodes is equal to half the wavelength of the stationary wave.
g) After time period $T/2$, particles having maximum displacement have minimum displacement and vice versa.

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শুভাশিষ চন্দ্র